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**İST.109 LİNEER CEBİR-I****FİNAL SINAV SORULARI****02.01.2020**

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| **SORU–1: (a)** $u=\left(\begin{matrix}-2,&-3,&1\end{matrix}\right)$**,** $v=\left(\begin{matrix}4,&2,&-3\end{matrix}\right)$ **ve** $s=\left(\begin{matrix}1,&-2,&3\end{matrix}\right)$ **vektörleri verilsin.** **(a)** $\left‖u-v+s\right‖^{2}$ **değerini (5P),** **(b)** $d\left(v, u-s\right)$ **değerini (5P),** **(c)** $ s×\left(u+v\right)$ **değerini (5P),** **(d)** $ \left(u∙v\right)s+$ $\left(v∙s\right)u$ **değerini (5P) ve** **(e)** $ u$ **ile** $v$ **arasındaki açıyı (5P) hesaplayınız.****SORU–2: (a)** $u=\left(\begin{matrix}1,&2,&4\end{matrix}\right)$ **vektörünü** $S=\left\{u\_{1}=\left(\begin{matrix}2,&-1,&1\end{matrix}\right), u\_{2}=\left(\begin{matrix}4,&-5&-1\end{matrix}\right) , u\_{3}=\left(\begin{matrix}1,&2,&2\end{matrix}\right)\right\}$**’deki vektörlerin bir lineer birleşimi olarak yazılabilirse yazınız (10 P).****(b)** $u=\left(\begin{matrix}a,&b,&c\end{matrix}\right)\in R^{3}$ **vektörlerinden oluşan;** $A=\left\{\left(\begin{matrix}a,&b,&c\end{matrix}\right): a+b-c=0\right\}$ **kümesinin** $R^{3}$ **vektör uzayının bir alt uzayı olup olmadığını gösteriniz (10 P).** **(c)** $u=\left(\begin{matrix}2,&1,&-2\end{matrix}\right) $ **ve** $v=\left(\begin{matrix}4,&2,&-3\end{matrix}\right) $**vektörlerine dik olan bir birim vektör bulunuz (5 P)** **SORU-3:** $A=\left[\begin{array}{c}\begin{matrix} 3&-1\\-1& 4\\ 2&-2\end{matrix} \begin{matrix}-3& 3\\ 1&-2\\ 1&-1\end{matrix}\\ -2 \begin{matrix}3& –2 2 \end{matrix}\end{array}\right]$ **matrisinin ikinci satıra göre determinantını hesaplayınız.** **SORU-4:** $A=\left[\begin{array}{c}\begin{matrix} 4&1&-2\\2&2& 1\\ 1&2& 2\end{matrix} \\ \end{array}\right]$ **matrisinin tersini hesaplayınız.**  |

**DİKKAT! AŞAĞIDAKİ UYARILARI OKUYUNUZ.**1. **Sorular eşit ve 25’şer puandır.**
2. **Bu sınavda verdiğiniz kâğıt sayısını sol üst köşeye yazmayı unutmayınız.**

**Başarılar Dilerim.****Prof. Dr. Kamil ALAKUŞ** |

**CEVAPLAR**

**CEVAP-1: (a)** $u-v+s=\left(-5,-7, 7\right)$$=>\left‖u-v+s\right‖^{2}=25+49+49=123$ **bulunur.**

 **(b)** $u-s=\left(-3,-1,-2\right) $$=>$$d\left(v, u-s\right)=\sqrt{\left[\left(4-\left(-3\right)\right)^{2}+\left(2-\left(-1\right)\right)^{2}+\left(-3-\left(-2\right)\right)^{2}\right]}$

$=\sqrt{49+9+1}$ **=**$\sqrt{59}$ **bulunur.**

**(c)** $u+v=\left(2, -1, -2\right)$$=>$$s×\left(u+v\right)=\left|\begin{matrix}1&-2&3\\2&-1&-2\end{matrix}\right|=\left(7, 8,3\right)$ **bulunur.**

**(d)** $\left(u\*v\right)=-17$ **ve** $\left(v\*s\right)=-9$$=>$$\left(u\*v\right)s+\left(v\*s\right)u=\left(1, 61, -60\right)$ **bulunur.**

**(e)** $Cosθ=\frac{\left(u.v\right)}{\left‖u\right‖ \left‖v\right‖}=\frac{-17}{\sqrt{14} \sqrt{29}}=-0,8437$$=>$$θ^{°}=2.575$ **bulunur.**

**CEVAP-2: (a)**$ xu\_{1}+yu\_{2}+zu\_{3}=u => x\left(2, -1, 1\right)+y\left(4, -5, -1\right)+z\left(1, 2, 2\right)=\left(1, 2, 4\right)$

$=>\left\{\begin{array}{c}2x+4y+z=1\\-x-5y+2z=2\\x-y+2z=4\end{array}\right.$ **denklem sistemi elde edilir. Bu sistem çözülürse**

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| **1/1/** | **2** | **4** | **1/** | **1** |
| **2/** | **-1** | **-5** | **2/** | **2** |
|  **-2/** | **1** | **-1** | **2/** | **4** |

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| **1/** | **0** | **-6** | **5/** | **5** |
| **1/** | **0** | **6** | **-3/** | **-7** |

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| **2** | **4** | **1/** | **1** |
| **0** | **-6** | **5/** | **5** |
| **0** | **0** | **2/** | **-2** |
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$2z=-2 =>z=-1$ **bulunur.** $y=-5/3$ **ve** $x=13/3$**elde edilir. Sonuç olarak;** $u=\frac{13}{3}u\_{1}-\frac{5}{3}u\_{2}-u\_{3}$ **olarak yazaılır.**

**(b) (*i)*** $A\ne ϕ$**’dir. (*ii)*** $u=\left(a\_{1},b\_{1},c\_{1}\right)$ **ve** $u=\left(a\_{2},b\_{2},c\_{2}\right)$$\in A$ **için** $\left(u+v\right)\in A$ **olmalı.** $=>u+v=\left(a\_{1}+a\_{2}, b\_{1}+b\_{2}, c\_{1}+c\_{2}\right)\in A$ **’dır. Zira** $\left(a\_{1}+a\_{2}\right)+ \left(b\_{1}+b\_{2}\right)-\left(c\_{1}+c\_{2}\right)=0$ **olup** $\left(u+v\right)$ **toplam vektörü de *A* kümesindedir. (*iii)*** $k\in R$ **için** $\left(ku\right)\in A$ **olmalı.** $=>$$ku=\left(ka\_{1}, kb\_{1}, kc\_{1}\right)\in A$**’dır. Çünkü** $ka\_{1},+kb\_{1},-kc\_{1}$**=0 olup bu vektör de *A* kümesindedir. Sonuç olarak; *A* kümesi** $R^{3}$**’ün bir alt vektör uzayıdır.**

**(c)** $s=u×v$ **vektörü hem** $u$**’ya ve hem de** $v$**’ye dik olduğuna göre önce** $s$ **vektörünü bulalım.** $s=u×v=\left(1, -2, 0\right)$ **bulunur. Bu vektörü normlarsak,** $s^{\*}=\frac{s}{\left‖s\right‖}$ **vektörü birim vektör olup hem** $u$**’ya ve hem de** $v$**’ye dik olur. Böylece aranan vektör,** $s^{\*}=\frac{1}{\sqrt{5}}\left(1, -2, 0\right)$ **bulunur.**

**CEVAP-3: İkinci satıra göre determinantı açalım.** $\left|A\right|=a\_{21}A\_{21}+a\_{22}A\_{22}+a\_{23}A\_{23}+a\_{24}A\_{24}$$=-A\_{21}+4A\_{22}+A\_{23}-2A\_{24}$$=-1\left(0\right)+4\left(0\right)+\left(-5\right)-2\left(-5\right)=5$ **bulunur.**

$$A\_{21}=(-1)^{2+1}\left|\begin{matrix}-1&-3&3\\-2&1&-1\\3&-2&2\end{matrix}\right|\begin{matrix}-1&-3\\-2&1\\3&-2\end{matrix}=-\left[-2+9+12-\left(9-2+12\right)\right]=0$$

$$A\_{22}=(-1)^{2+2}\left|\begin{matrix}3&-3&3\\2&1&-1\\-2&-2&2\end{matrix}\right|\begin{matrix}3&-3\\2&1\\-2&-2\end{matrix}=\left[6-6-12—\left(-6+6-12\right)\right]=0$$

$$A\_{23}=(-1)^{2+3}\left|\begin{matrix}3&-1&3\\2&-2&-1\\-2&3&2\end{matrix}\right|\begin{matrix}3&-1\\2&-2\\-2&3\end{matrix}=-\left[-12-2+18—\left(12-9-4\right)\right]=-5$$

$$A\_{24}=(-1)^{2+4}\left|\begin{matrix}3&-1&-3\\2&-2&1\\-2&3&-2\end{matrix}\right|\begin{matrix}3&-1\\2&-2\\-2&3\end{matrix}=\left[12+2-18—\left(-12+9+4\right)\right]=-5$$

**CEVAP-4:**

**Yol-1: Gauss indirgeme ve Kanonik Form ile çözüm: Önce ek matrisi oluşturalım.** $M=\left[A / I\right]$

$$\begin{matrix}1/1\\-2\\-4/\end{matrix}/\left[\begin{matrix}4&1&-2\\2&2&1\\1&2&2\end{matrix} \begin{matrix}/\\/\\/\end{matrix} \begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}\right]\~\left[\begin{matrix}4&1&-2\\0&-3&-4\\0&-7&-10\end{matrix} \begin{matrix}/\\/\\/\end{matrix} \begin{matrix}1&0&0\\1&-2&0\\1&0&-4\end{matrix}\right]\~\begin{matrix}\\7/\\-3/\end{matrix}\left[\begin{matrix}4&1&-2\\0&-3&-4\\0&0&2\end{matrix} \begin{matrix}/\\/\\/\end{matrix} \begin{matrix}1&0&0\\1&-2&0\\4&-14&12\end{matrix}\right]$$

$$\~\begin{matrix}\\\\\left(\frac{1}{2}\right)/\end{matrix}\left[\begin{matrix}4&1&-2\\0&-3&-4\\0&0&2\end{matrix} \begin{matrix}/\\/\\/\end{matrix} \begin{matrix}1&0&0\\1&-2&0\\4&-14&12\end{matrix}\right]\~\begin{matrix}1/\\1/\\2/4\end{matrix}\left[\begin{matrix}4&1&-2\\0&-3&-4\\0&0&1\end{matrix} \begin{matrix}/\\/\\/\end{matrix} \begin{matrix}1&0&0\\1&-2&0\\2&-7&6\end{matrix}\right]\~(-\begin{matrix}\\\frac{1}{3}\\\end{matrix})/\left[\begin{matrix}4&1&0\\0&-3&0\\0&0&1\end{matrix} \begin{matrix}/\\/\\/\end{matrix} \begin{matrix}5&-14&12\\9&-30&24\\2&-7&6\end{matrix}\right]$$

$$\~\begin{matrix}1/\\-1/\\\end{matrix}\left[\begin{matrix}4&1&0\\0&1&0\\0&0&1\end{matrix} \begin{matrix}/\\/\\/\end{matrix} \begin{matrix}5&-14&12\\-3&10&-8\\2&-7&6\end{matrix}\right]\~\begin{matrix}\frac{1}{4}/\\\\\end{matrix}\left[\begin{matrix}4&0&0\\0&1&0\\0&0&1\end{matrix} \begin{matrix}/\\/\\/\end{matrix} \begin{matrix}8&-24&20\\-3&10&-8\\2&-7&6\end{matrix}\right]\~\left[\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix} \begin{matrix}/\\/\\/\end{matrix} \begin{matrix}2&-6&5\\-3&10&-8\\2&-7&6\end{matrix}\right]$$

$=>$ $A^{-1}=\left[\begin{matrix}2&-6&5\\-3&10&-8\\2&-7&6\end{matrix} \right]$ **bulunur.**

**Yol-2: Determinantlarla çözüm:** $\left|A\right|=1\ne 0$ **olduğundan** $A$ **matrisinin tersi vardır.** $A^{-1}=\frac{1}{\left|A\right|}Adj(A)$ **yazılır.**

**Kofaktörler:**

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| $$A\_{11}=2; $$ | $$A\_{12}=-\left(3\right)=-3; $$ | $A\_{13}=2$**;**  | $$A\_{21}=-\left(6\right)=-6; $$ | $A\_{22}=10$**;**  |
| $$A\_{23}=-\left(7\right)=-7; $$ | $A\_{31}=5$**;**  | $A\_{32}$**=-(8)=-8;**  | $$A\_{33}=6$$ |  |

**bulunur. Böylece Adj(A) matrisi,** $Adj\left(A\right)=\left[\begin{matrix}2&-3&2\\-6&10&-7\\5&-8&6\end{matrix}\right]^{t}=\left[\begin{matrix}2&-6&5\\-3&10&-8\\2&-7&6\end{matrix}\right]$ **olup,** $A$ **matrisinin tersi,** $A^{-1}=\left[\begin{matrix}2&-6&5\\-3&10&-8\\2&-7&6\end{matrix} \right]$ **bulunur.**